

# 3D Printing in Topology

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# Topics.

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- Theory
- Generating 3D Models
- 3D Printing
- Results and Future Research
- Final Remarks and Questions

# Theory.

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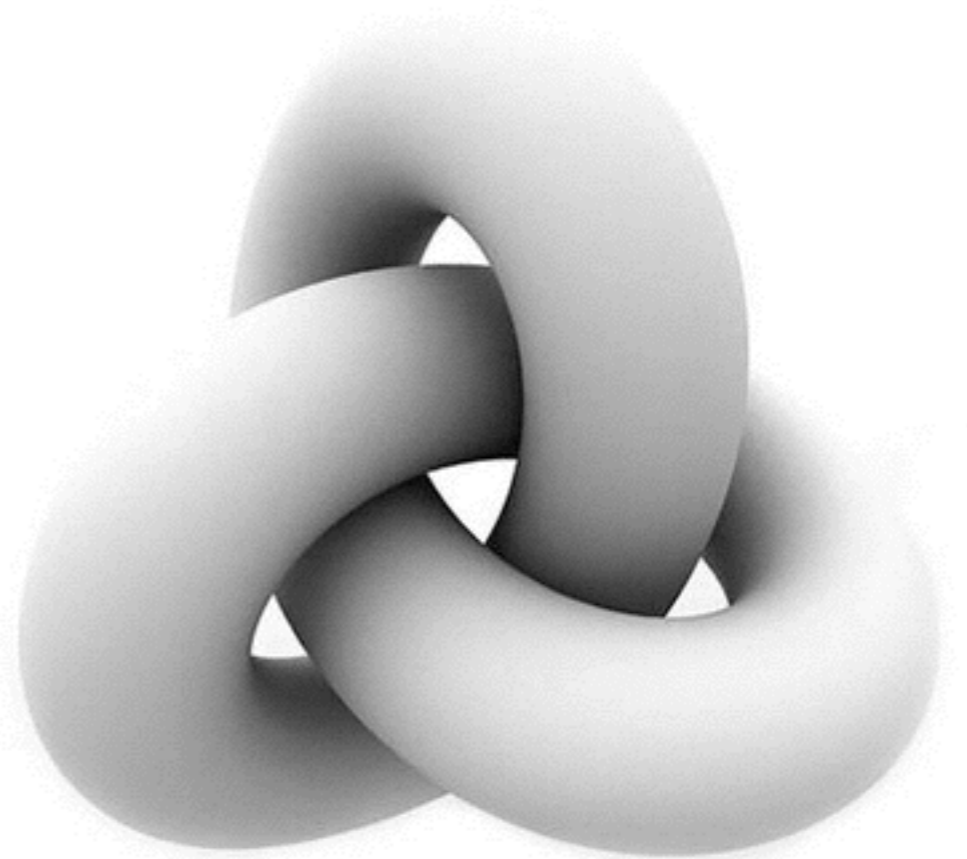
- Consider a function

$$\mathbb{R}^4 = \mathbb{C}^2 \rightarrow \mathbb{C} = \mathbb{R}^2$$

$$(u, v) \rightarrow u^2 - v^3$$

- This function's zero-set generates a **trefoil knot** through infinity
- Construction: take the inverse image of sets in  $\mathbb{C} = \mathbb{R}^2$
- Example:

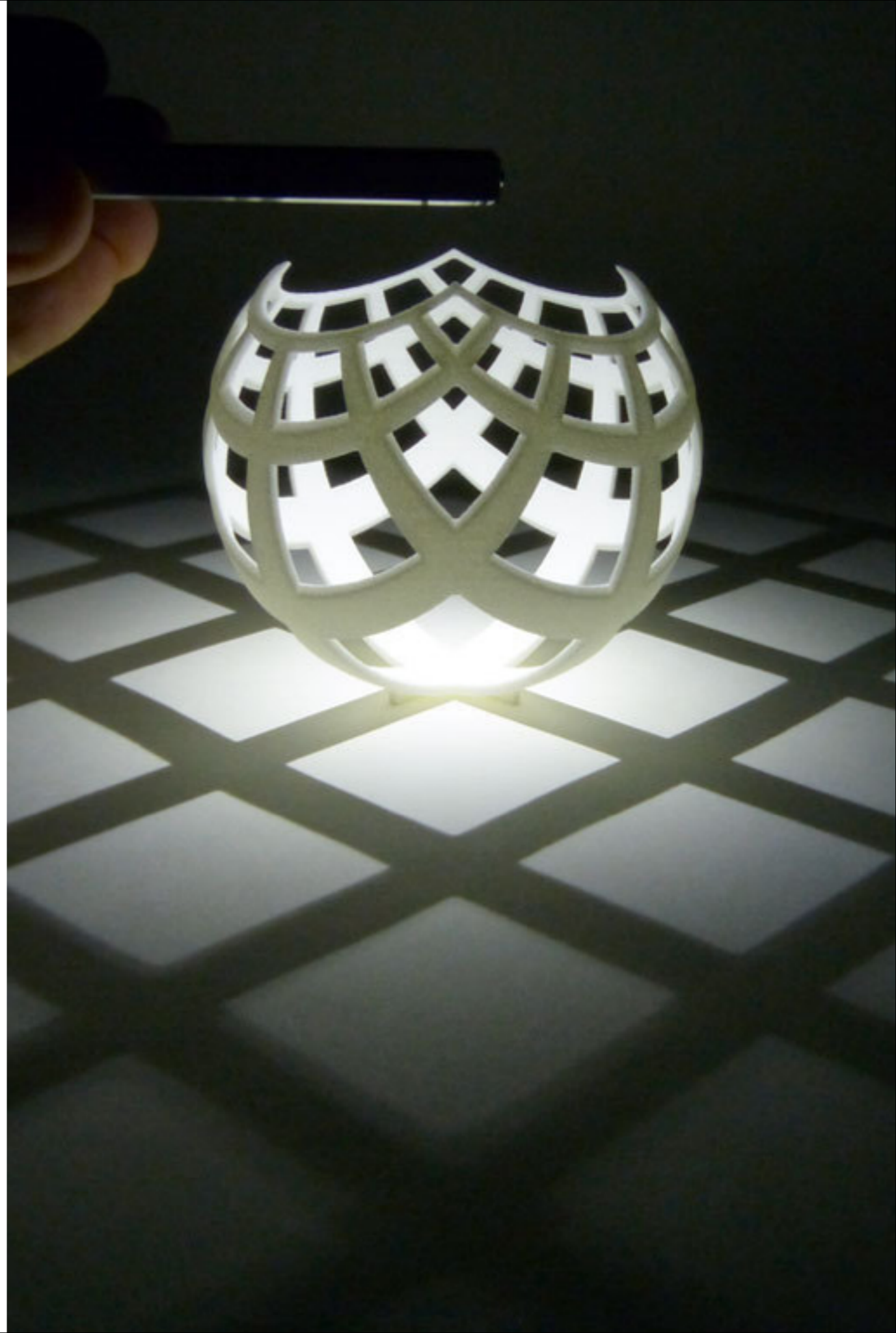
$$P^{-1}(\{0\}) = \{(u, v) | u^2 = v^3\}$$

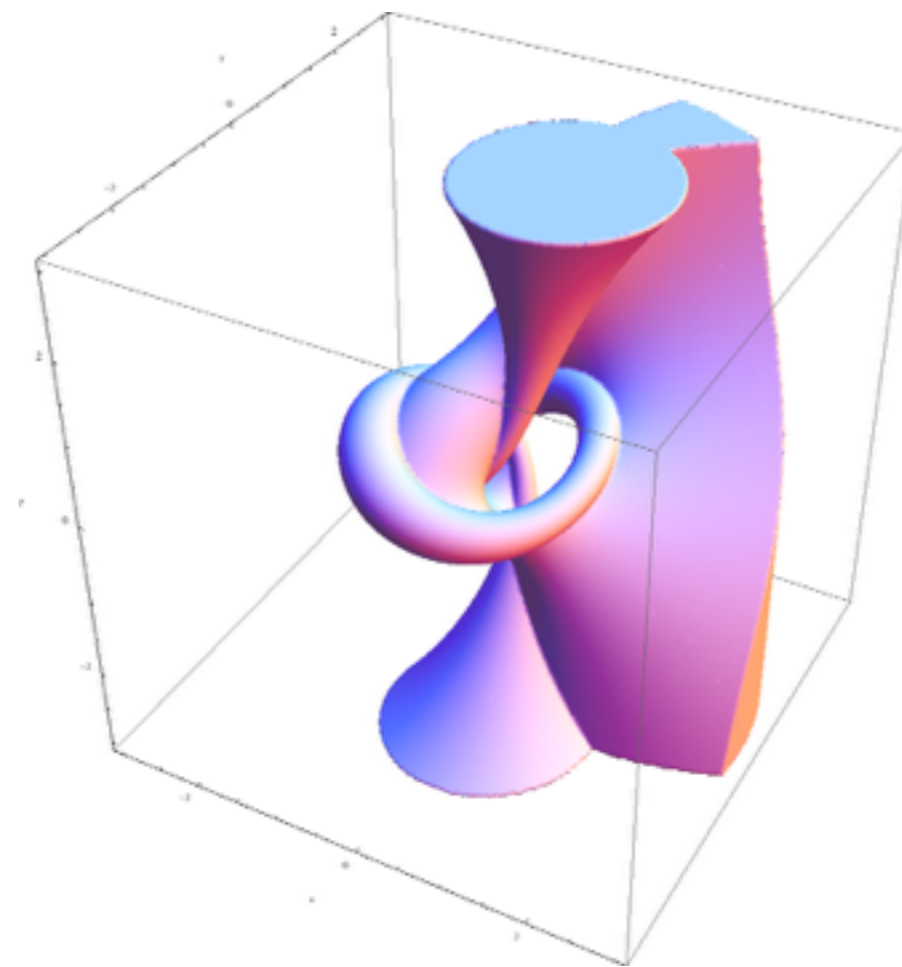
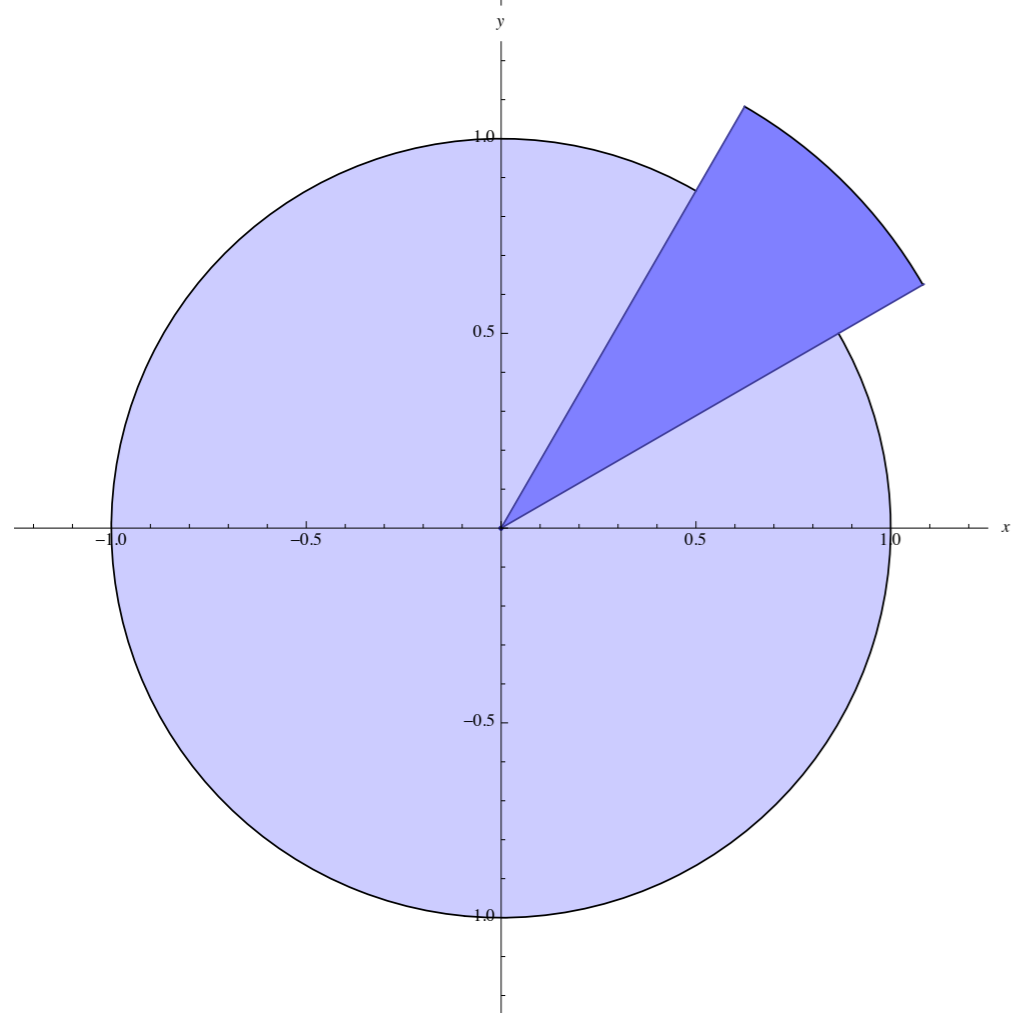
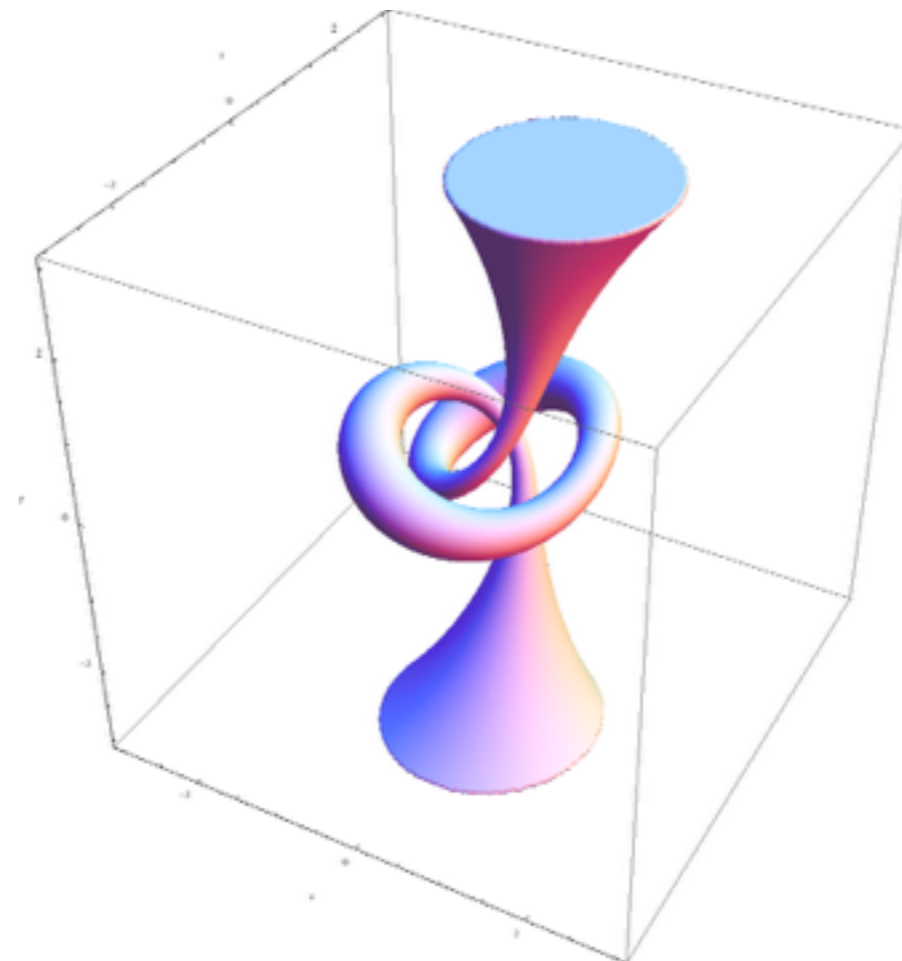
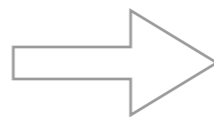
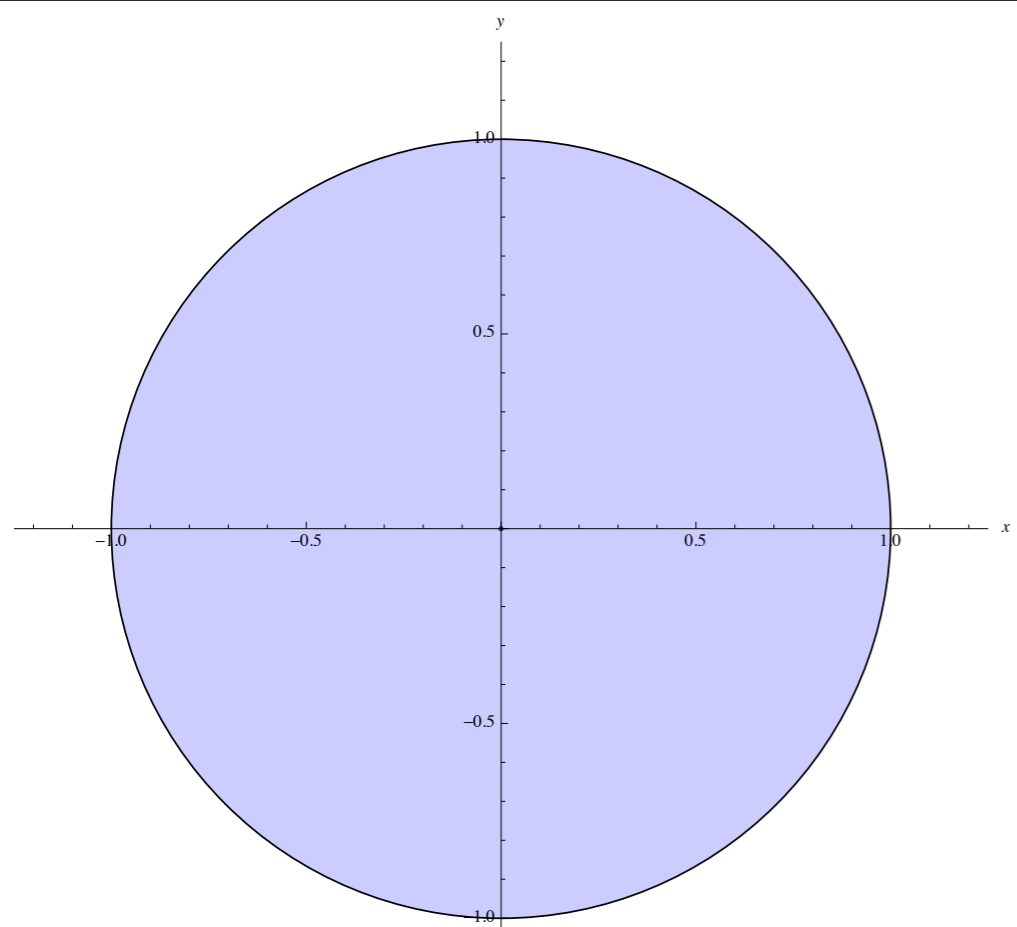


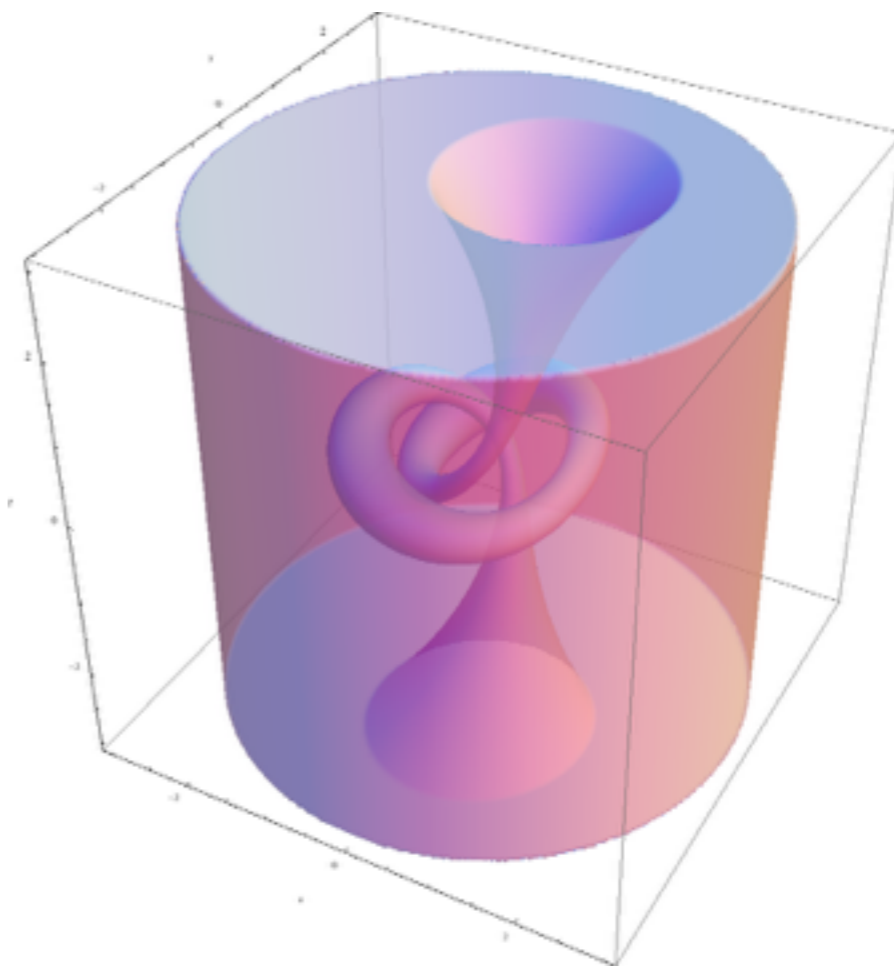
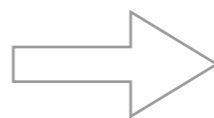
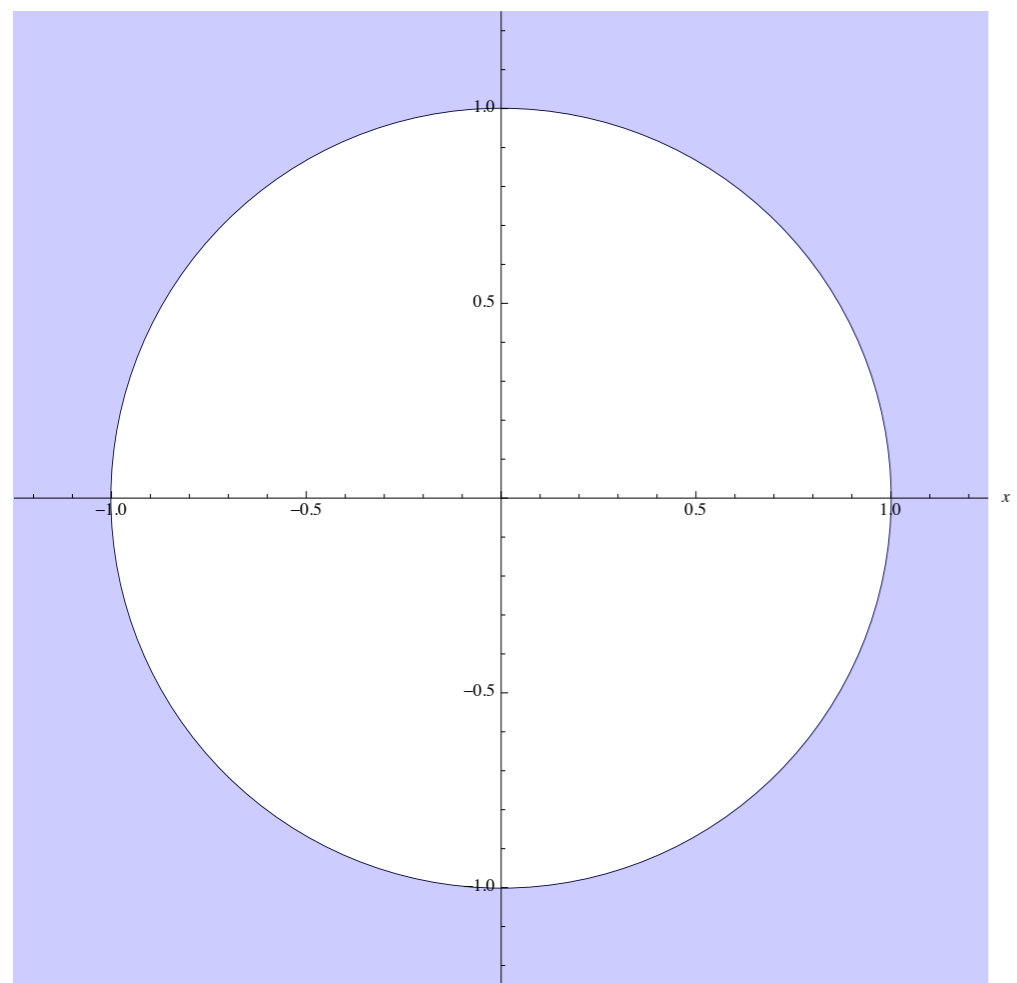
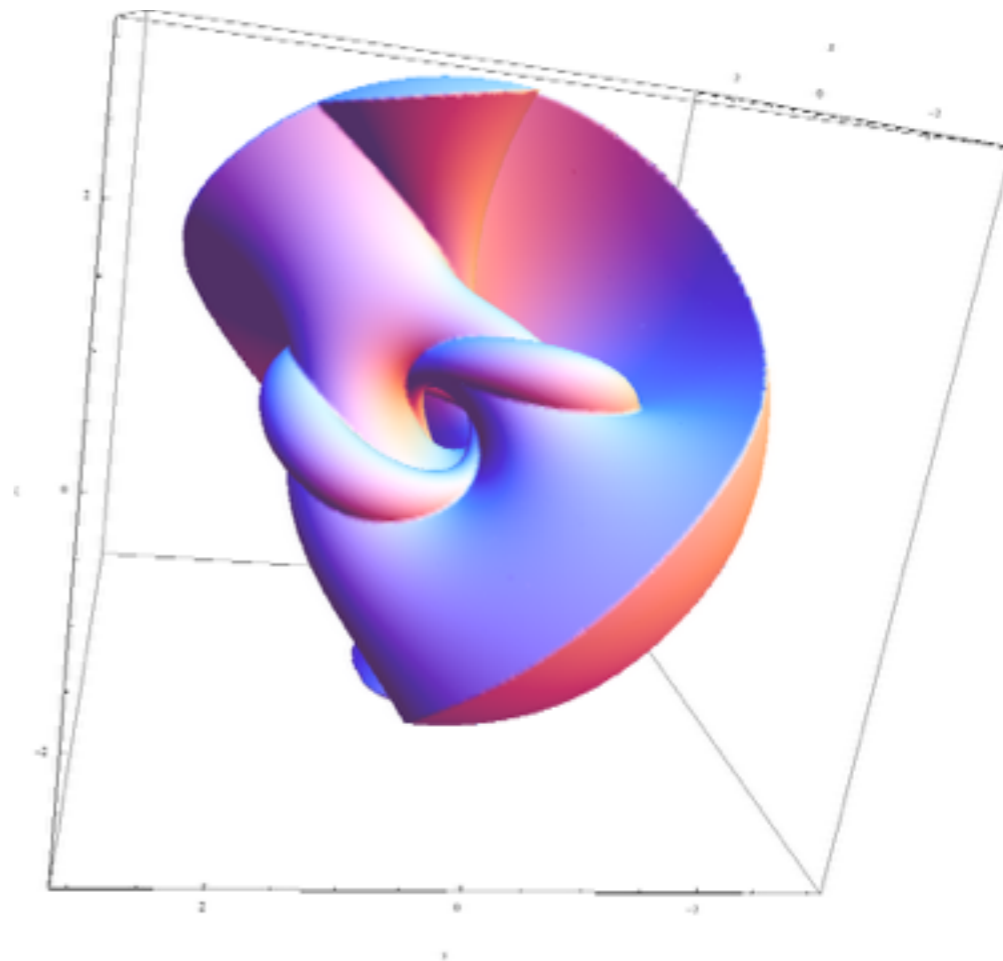
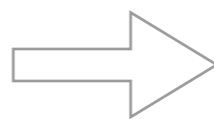
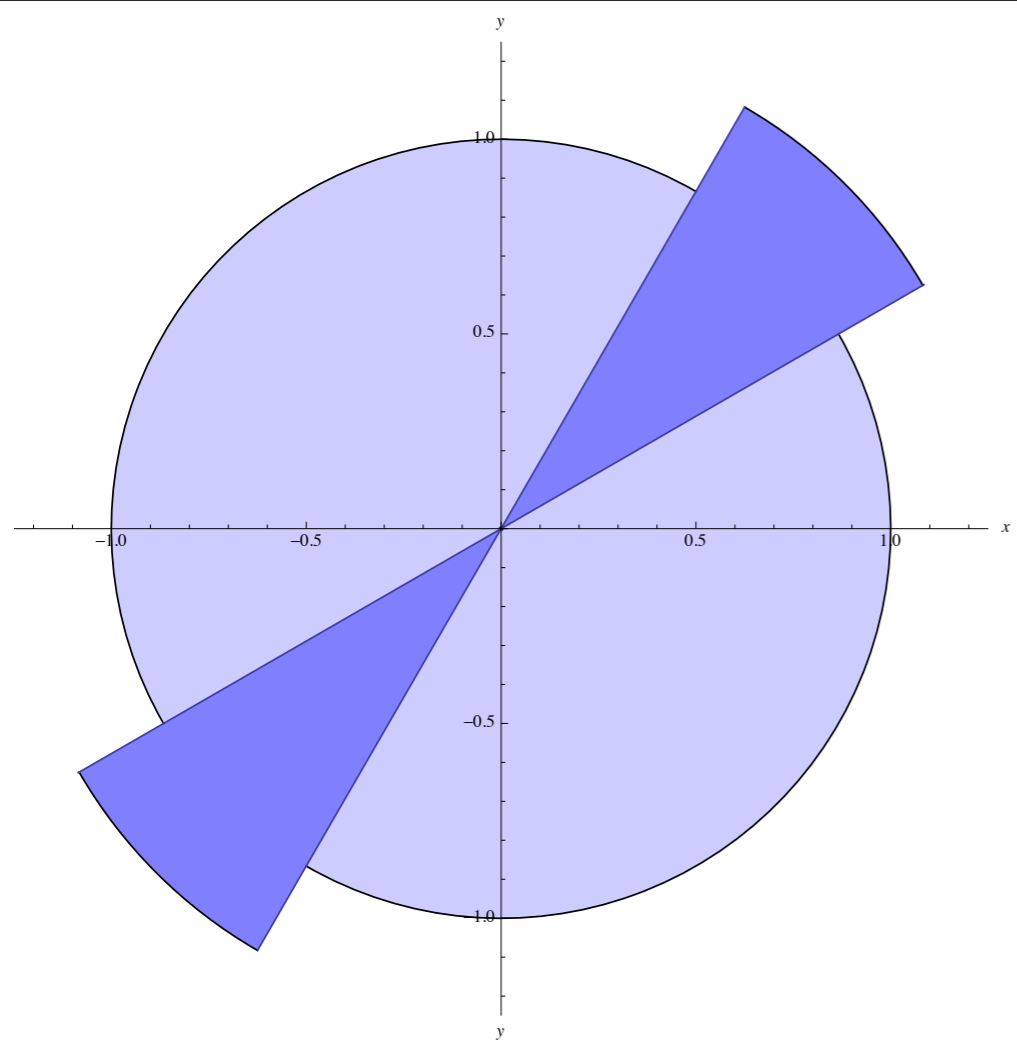
# Theory.

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- **Stereographic Projection:**  
Mapping from the sphere in 3D to the plane in 2D
- Now generalize: project from the “sphere” in 4D ( $\mathbb{C}^2 = \mathbb{R}^4$ ) onto  $\mathbb{R}^3$
- **Open-book decomposition:**  
imagine our knot is the spine of book, what does a page look like inside?





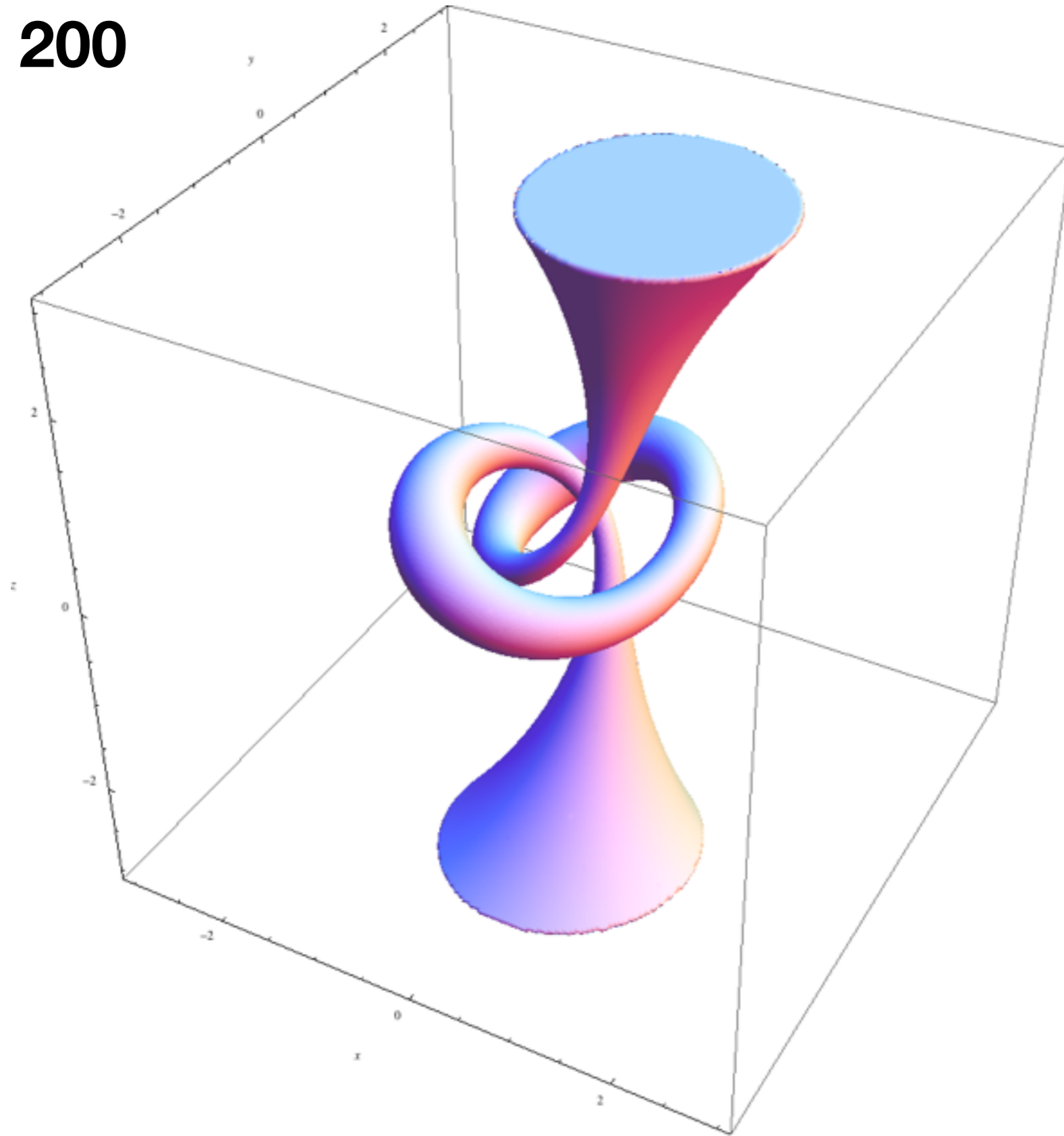


# Generating 3D Models.

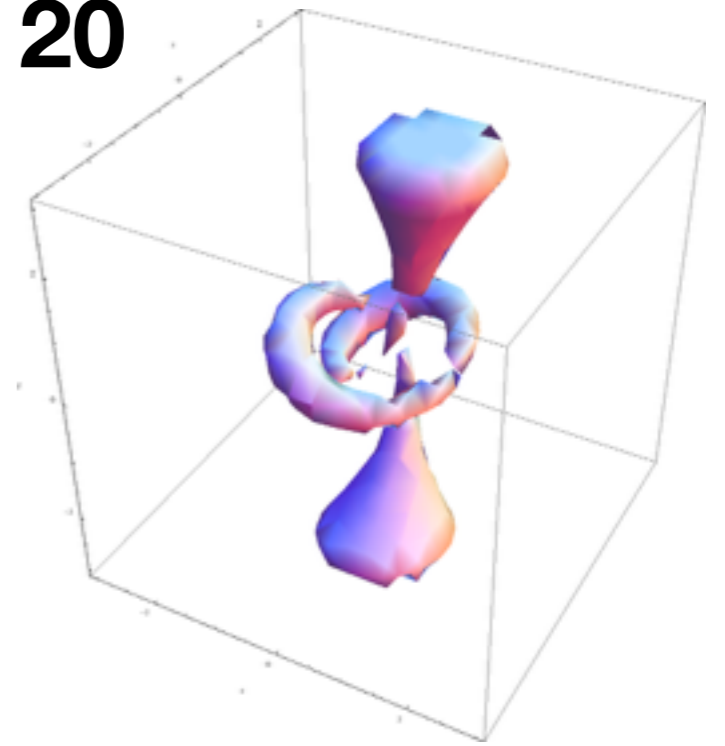
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- **STereoLithography** files (.stl) describe surface geometry of 3D objects
- Software: CAD, Mathematica, Meshlab, Blender, etc.
- **Resolution:** quality of model
  - more triangles + more vertices = higher resolution
  - Achieved in Mathematica via PlotPoints

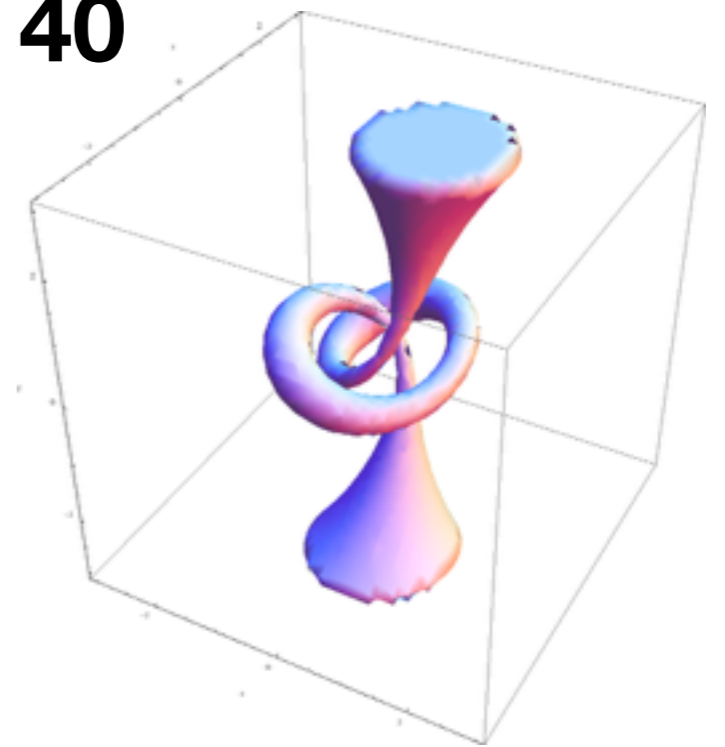
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**20**



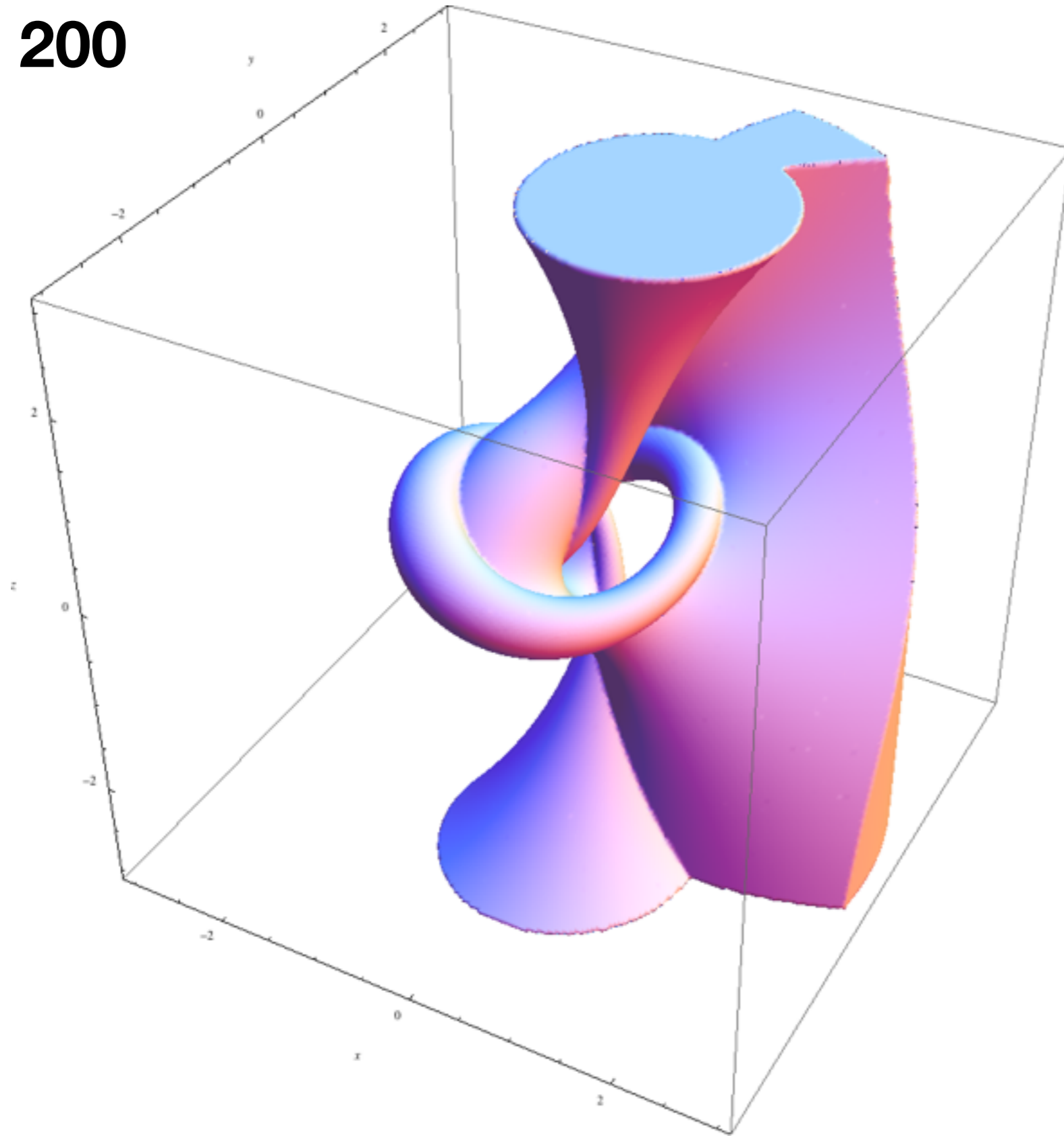
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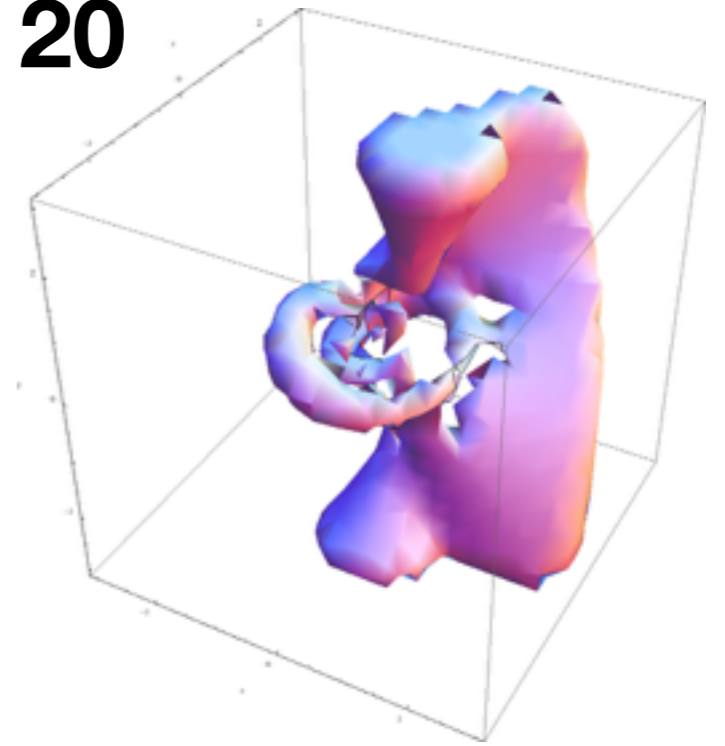
Generating 3D Models—Trefoil Knot.  
Created in Mathematica.



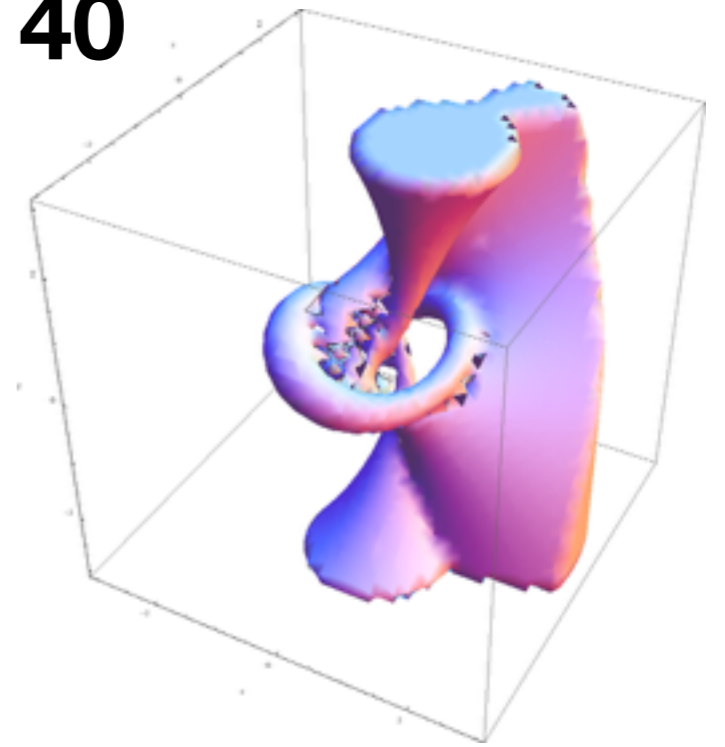
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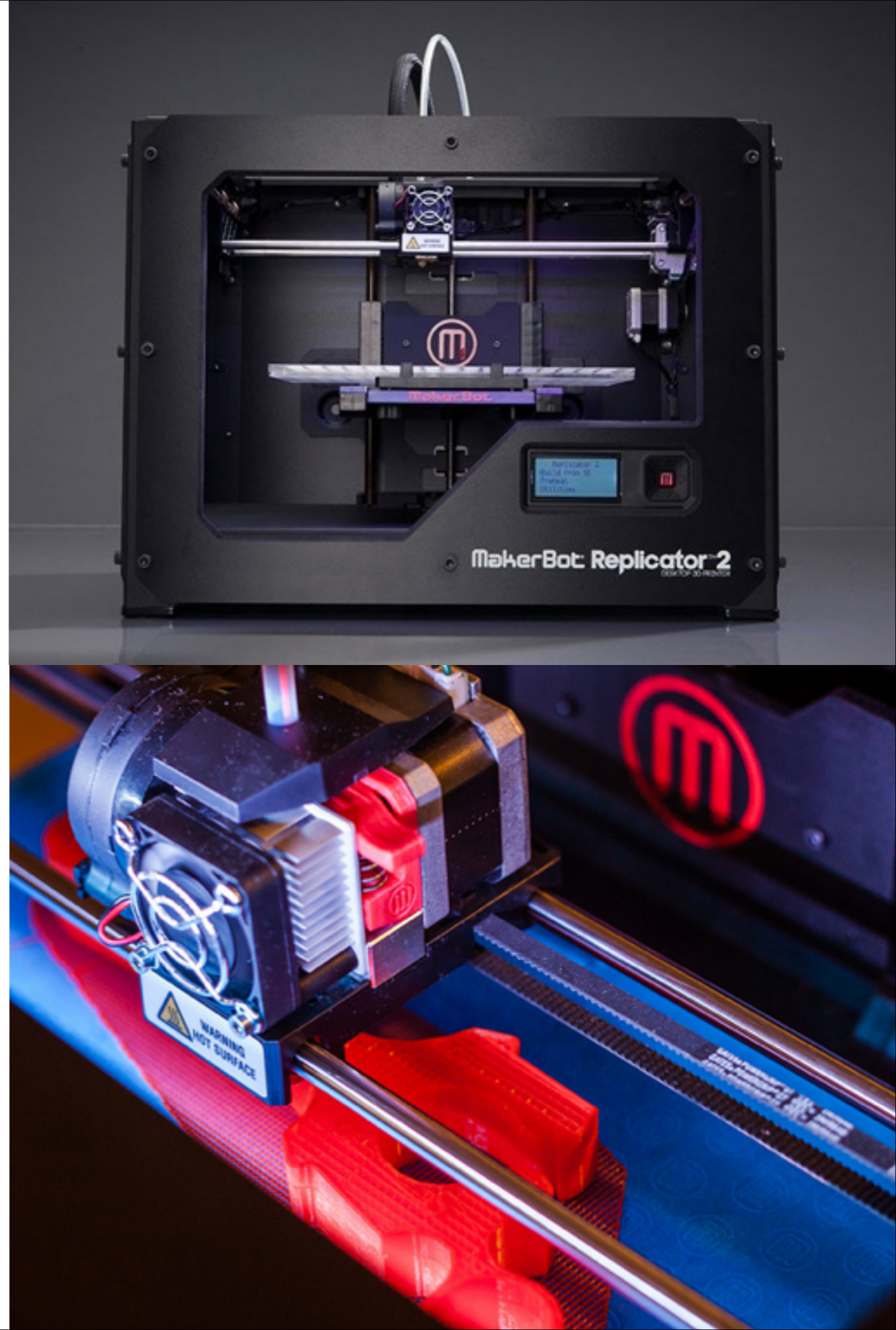


Generating 3D Models—Trefoil Knot + Page.  
Created in Mathematica.

# 3D Printing.

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- “Process of making a three-dimensional solid object of virtually any shape from a digital model.” -Wikipedia
- **Makerbot Replicator 2**
- **Additive Layering:** printer extrudes melted plastic in thin (100 micron, 0.1 millimeter) layers and builds object from



# 3D Printing.

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- Controls:
  - **Rafts** and **Supports**
  - Infill percentage (how solid/hollow the object is)
  - Layer Height (100 microns, 0.1 millimeters)
  - Extruder temperature ( $\sim 230^{\circ}$  C,  $\sim 446^{\circ}$  F)
  - Extruder speed when printing/traveling (90 mm/s, 150 mm/s)

# Results and Future Research.

- Posted on **Thingiverse**
  - Makerbot's website for digital design file sharing
  - Currently has \_\_\_ views and \_\_\_ downloads
- Determining best way to cut pages to build knots
- Print more complicated knots with more pages

The screenshot shows a Thingiverse page for a 3D printed object titled "Trefoil Knot" by user fredhohman, published on Feb 7, 2014. The main image features a red 3D printed trefoil knot on a desk next to a laptop. A secondary image shows a computer screen displaying a 3D model of the knot. The page includes a navigation bar with "DASHBOARD", "EXPLORE", and "CREATE" options, and a search bar. On the right, there are statistics: 14 likes, 16 collections, 0 comments, 0 "I Made One"s, and 0 remixes. A prominent blue button says "Download This Thing!". Below the main image is a tabbed interface with "Thing Info" selected. The description states: "This is a variation of the **Trefoil Knot** that I made for my undergraduate research project. The .stl file was made using Mathematica's RegionPlot3D with a PlotPoints of 200 (a fairly high resolution). Read more about about the trefoil knot on [Wikipedia](#). Read more about my projects on [my website](#)." Below the description is a "More from Math" section with a "view more >" link and several thumbnail images of other 3D printed mathematical objects. On the right side, there are additional statistics: 774 views and 133 downloads, a "Found in Math" tag, a "Report Thing as Inappropriate" link, a "Liked By" section with user avatars, a "Tags" section with tags like "3D\_printing", "knot", "math", "mathematica", "topology", and "trefoil\_knot", and a "License" section showing the Creative Commons Attribution-ShareAlike license.

# Final Remarks and Questions.

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- **Tinkercad:** free 3D modeling program that runs in the browser (Chrome)—no downloading required!
  - Short tutorial (~10 minutes)
  - Easiest way to begin to model
- **Thingiverse**
- 3D printing technology is rapidly growing

# Final Remarks and Questions.

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Questions?

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**References.** (online)

- Mandalland Blogspot—  
Triangles
- Wikipedia—3D Printing
- Thingiverse—Stereographic  
Projection
- Makerbot
- Atomic Spin—Replicator 2  
Issues