3D Printing in Topology

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Topics.

- Theory
- Generating 3D Models
- 3D Printing
- Results and Future Research
- Final Remarks and Questions

Theory.

- Consider a function

$$\mathbb{R}^4 = \mathbb{C}^2 \to \mathbb{C} = \mathbb{R}^2$$

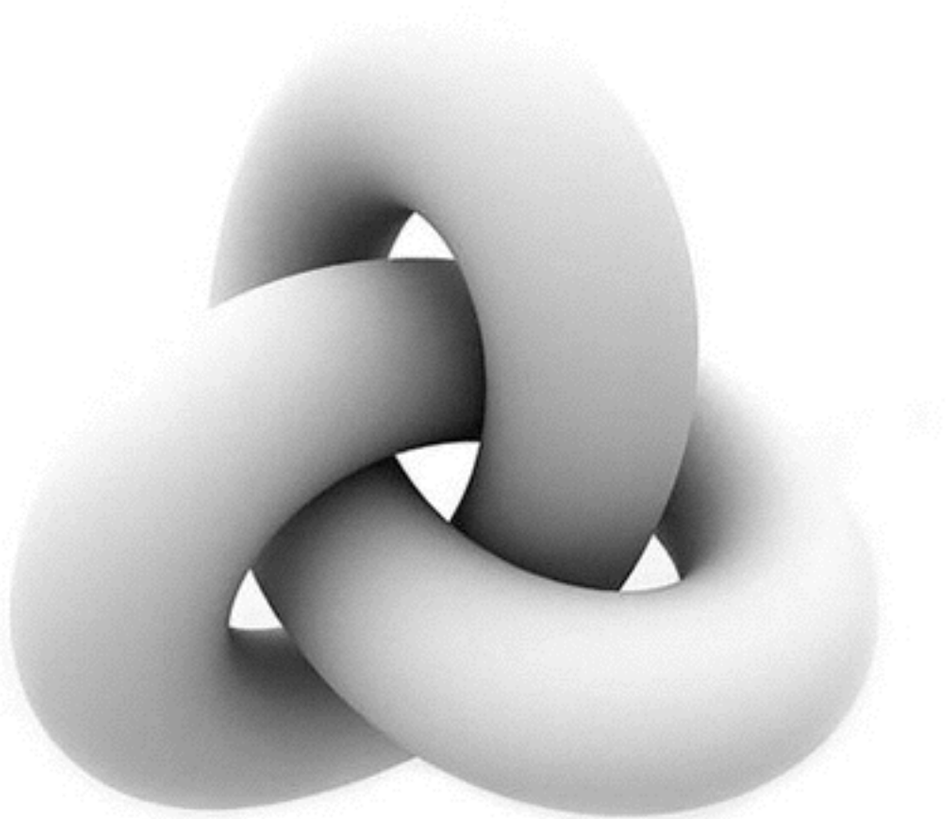
$$(u, v) \to u^2 - v^3$$

- This function's zero-set generates a **trefoil knot** through infinity

- Construction: take the inverse image of sets in $\mathbb{C} = \mathbb{R}^2$

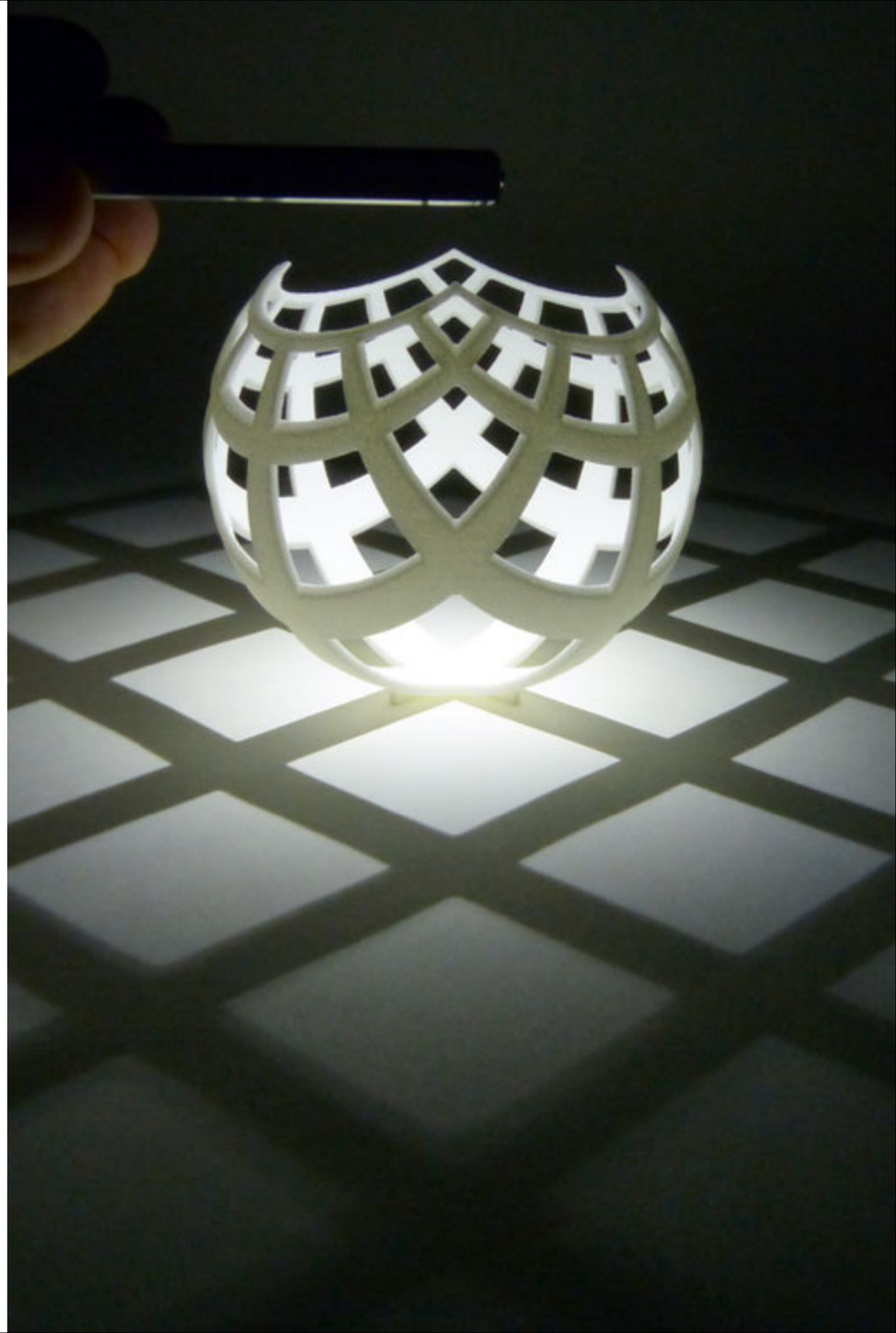
- Example:

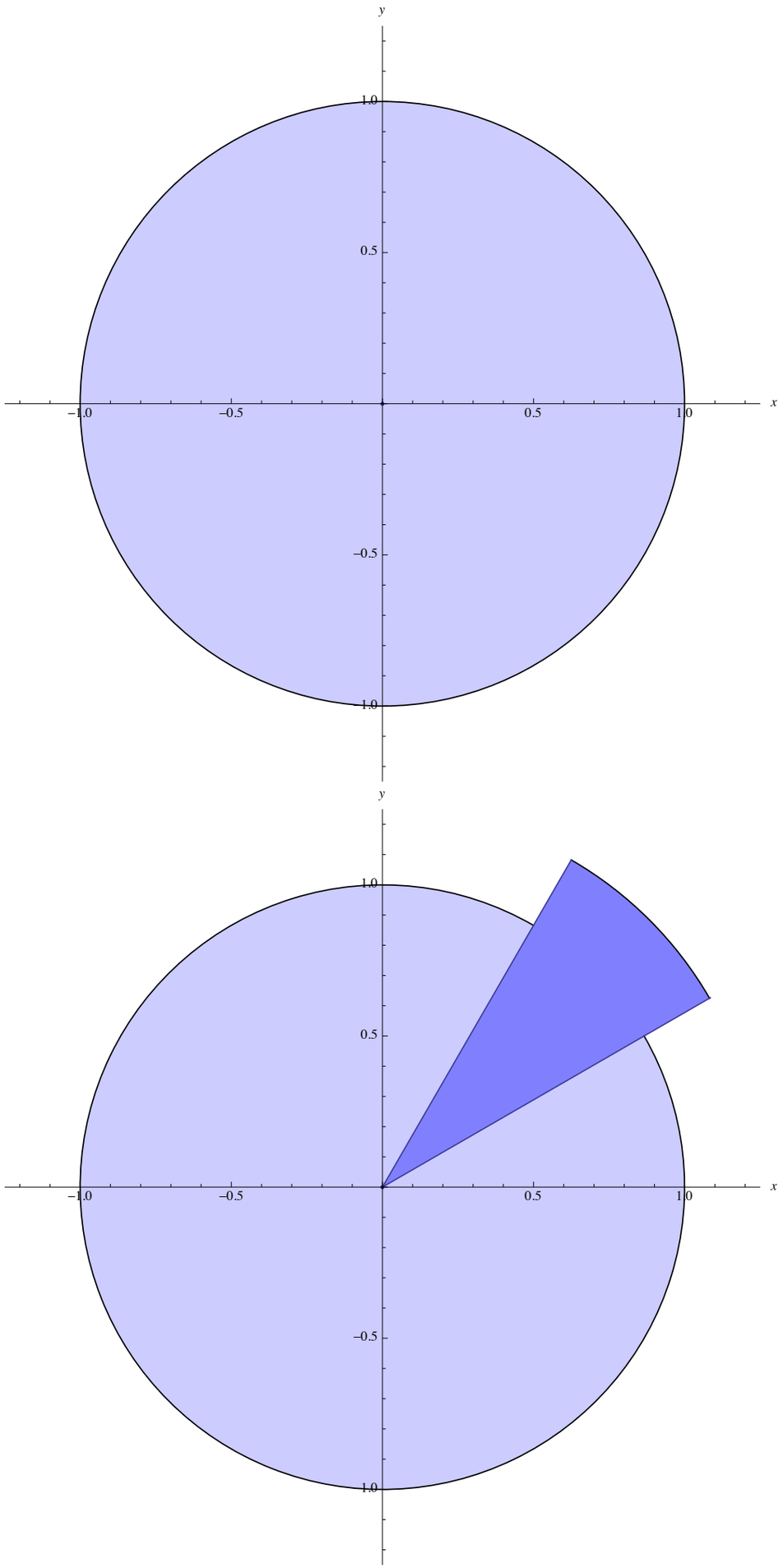
$$P^{-1}(\{0\}) = \{(u, v) | u^2 = v^3\}$$



Theory.

- **Stereographic Projection**: Mapping from the sphere in 3D to the plane in 2D
- Now generalize: project from the "sphere" in 4D ($\mathbb{C}^2 = \mathbb{R}^4$) onto $\mathbb{R}^3$
- **Open-book decomposition**: imagine our knot is the spine of book, what does a page look like inside?

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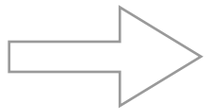

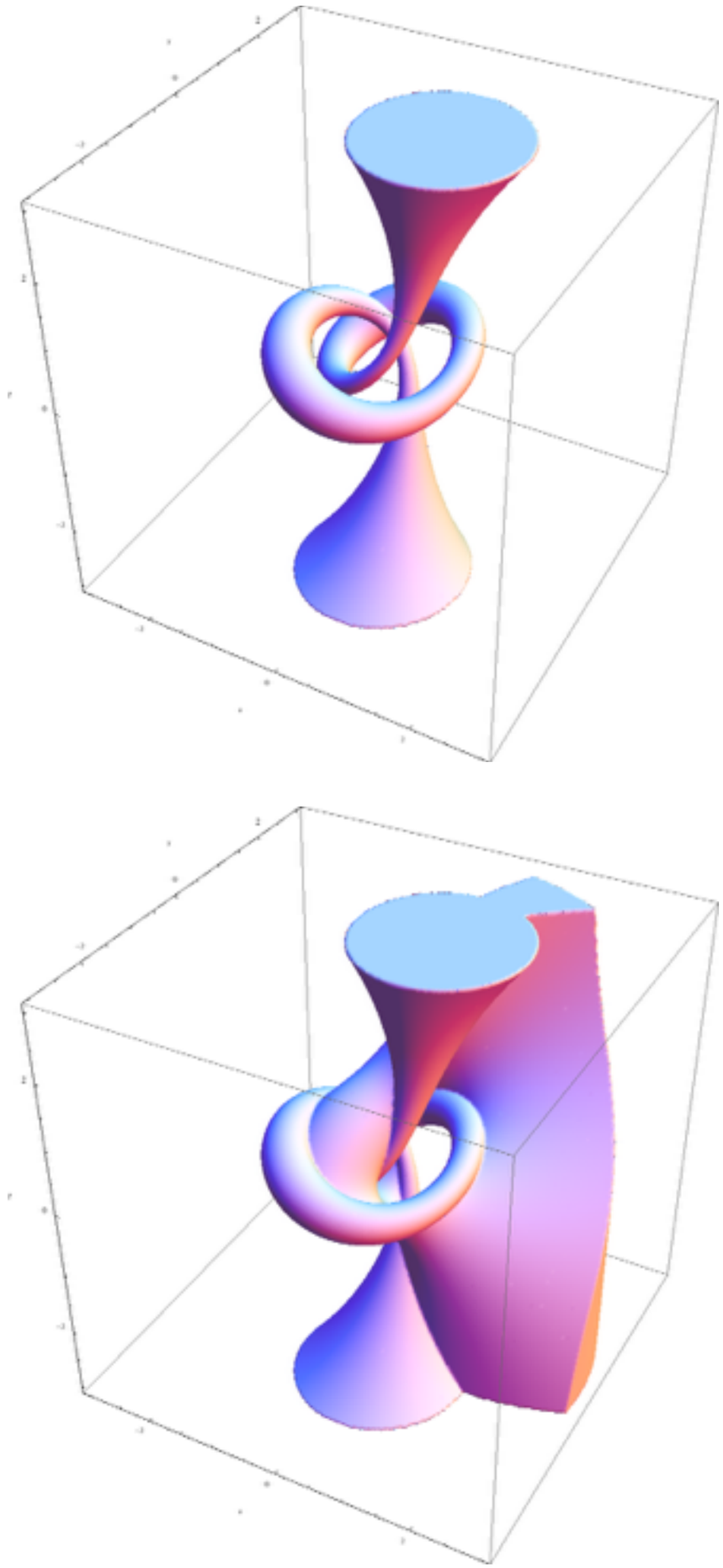

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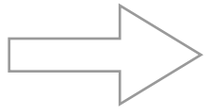

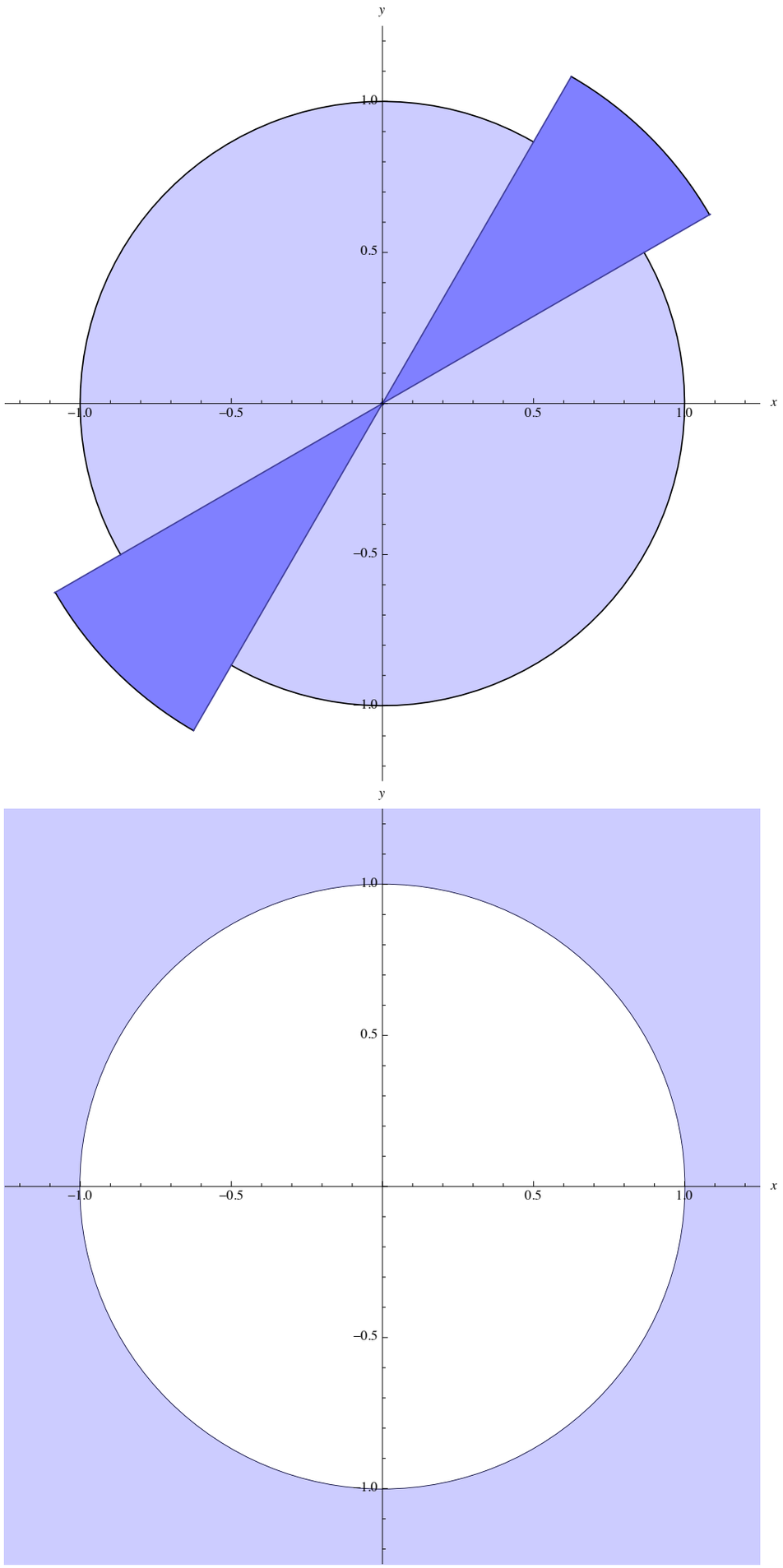
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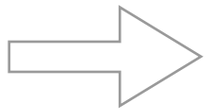

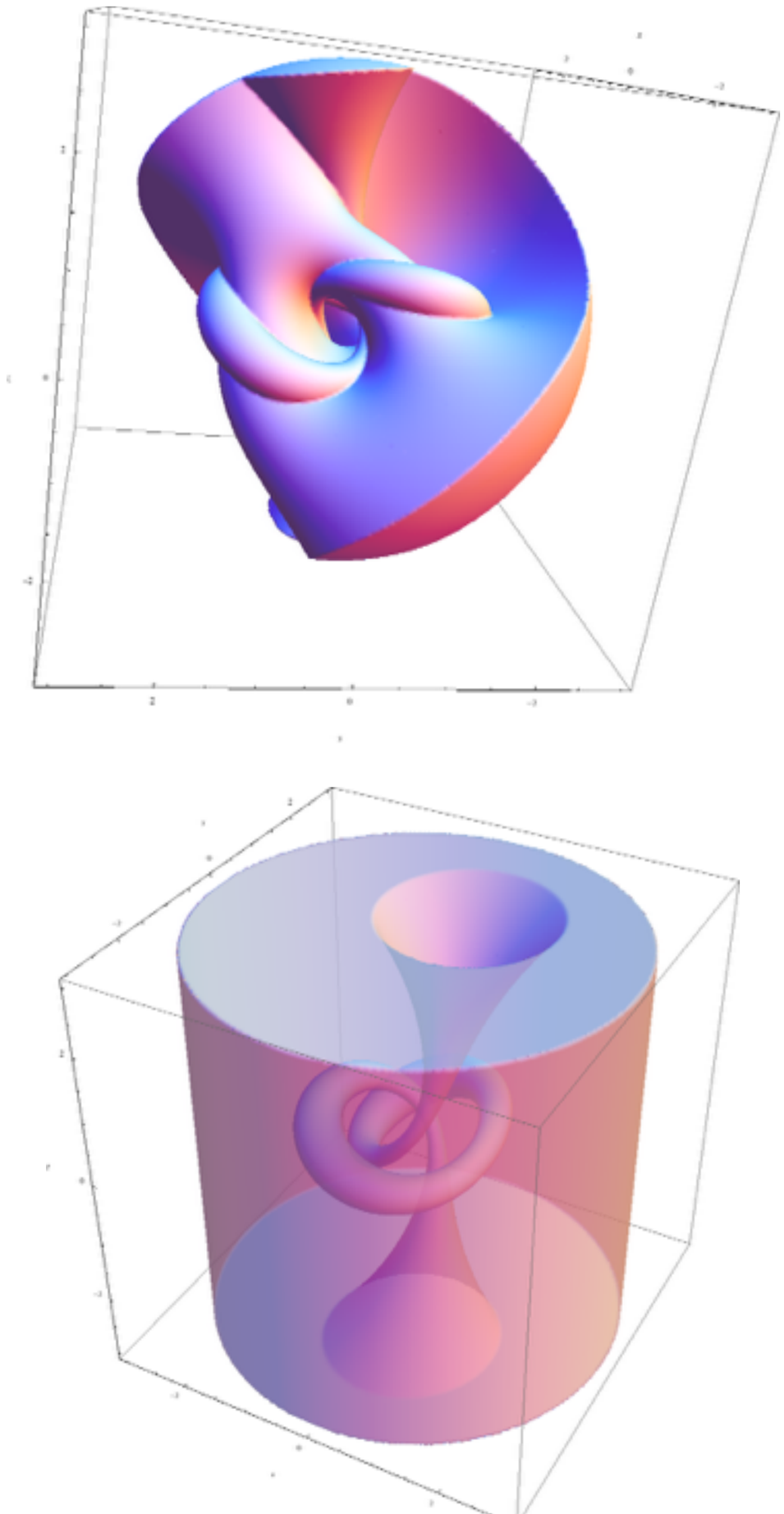

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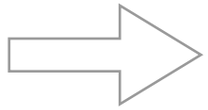

Generating 3D Models.

- **ST**ereo**L**ithography files (.stl) describe surface geometry of 3D objects
- Software: CAD, Mathematica, Meshlab, Blender, etc.
- **Resolution**: quality of model
  - more triangles + more vertices = higher resolution
  - Achieved in Mathematica via PlotPoints

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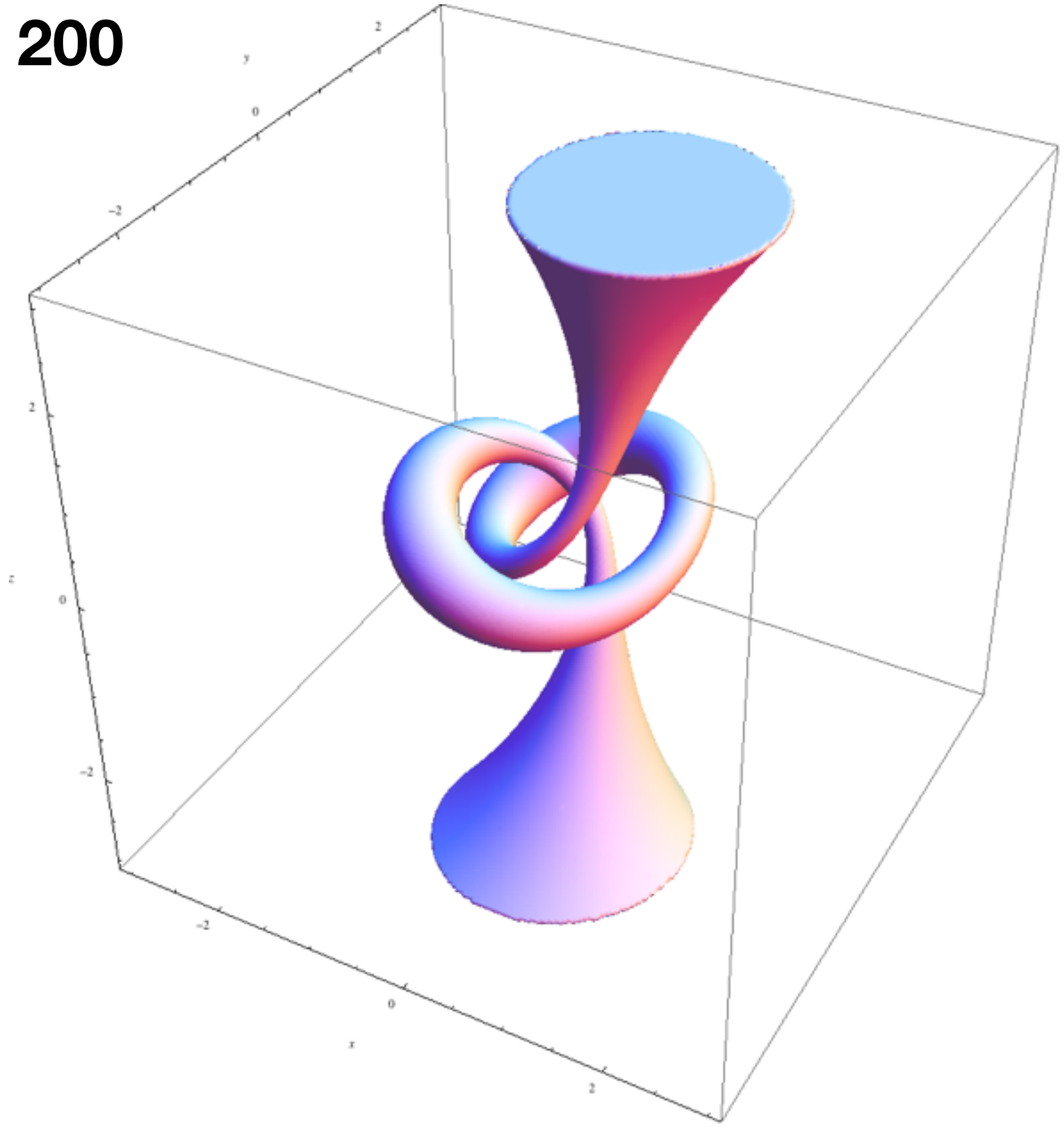

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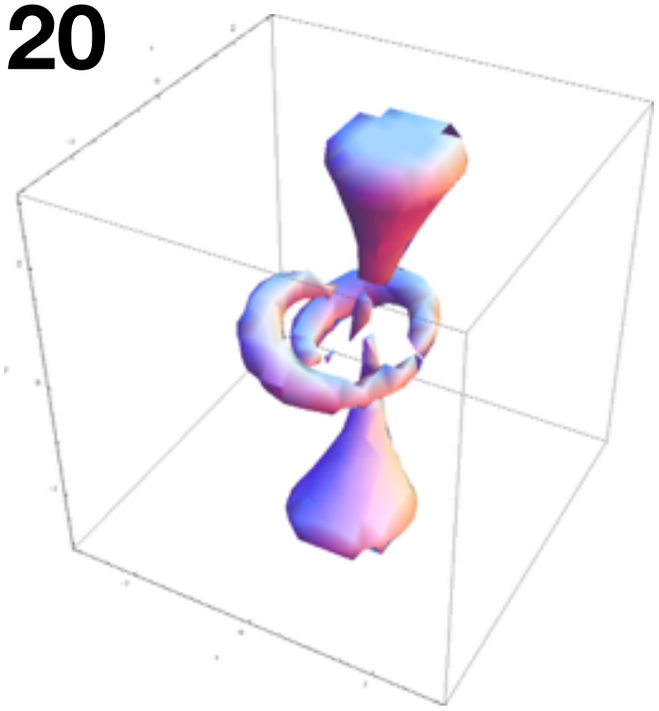

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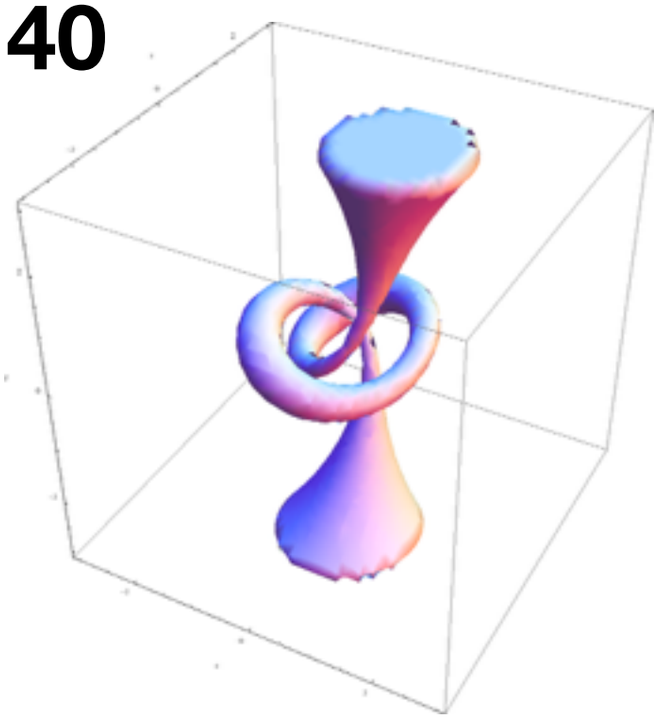

Generating 3D Models—Trefoil Knot.
Created in Mathematica.

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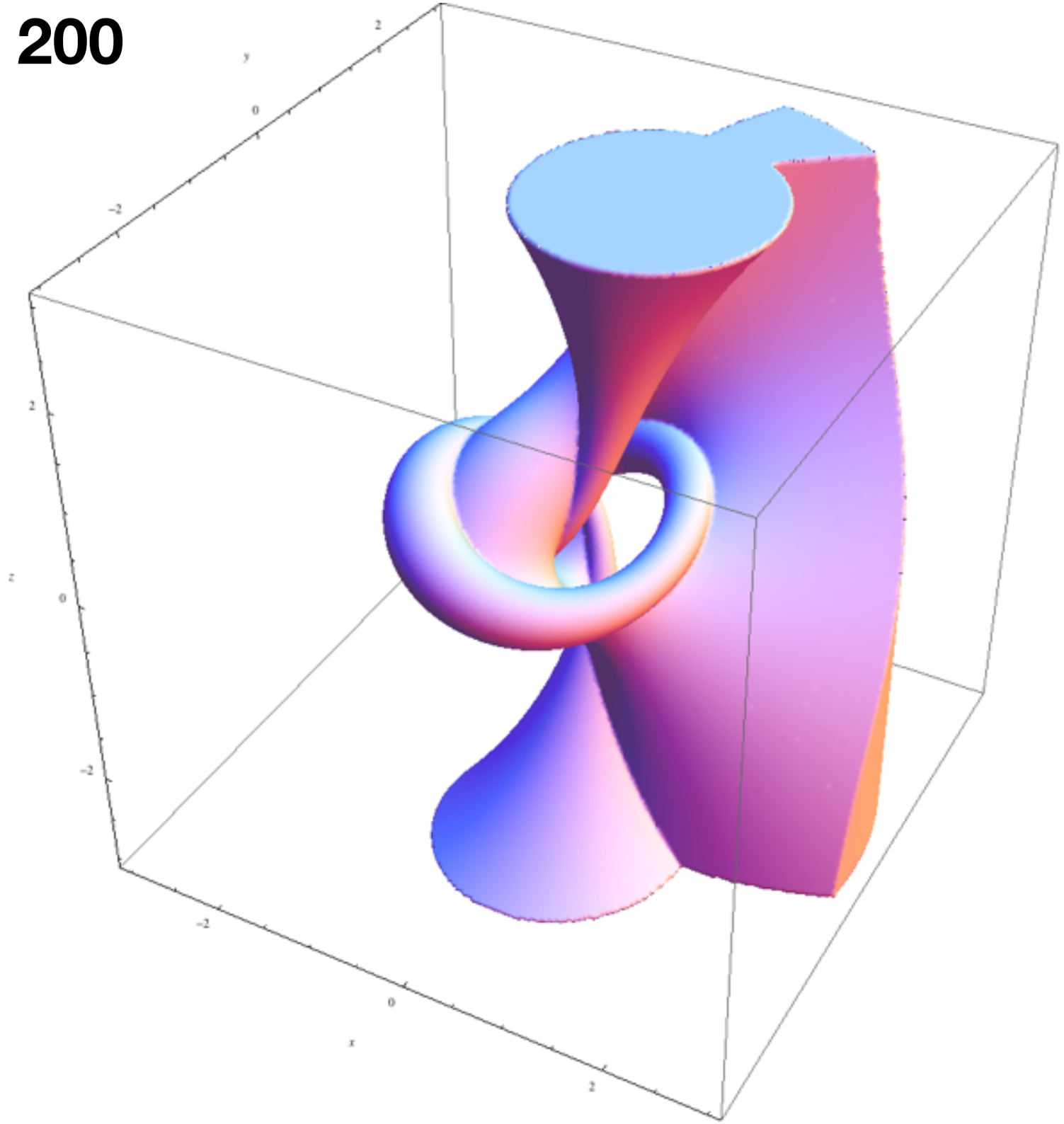

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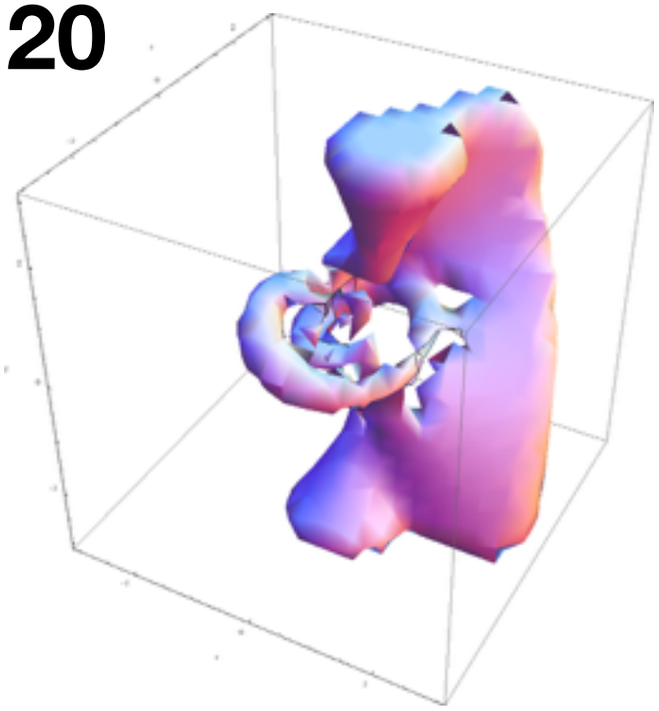

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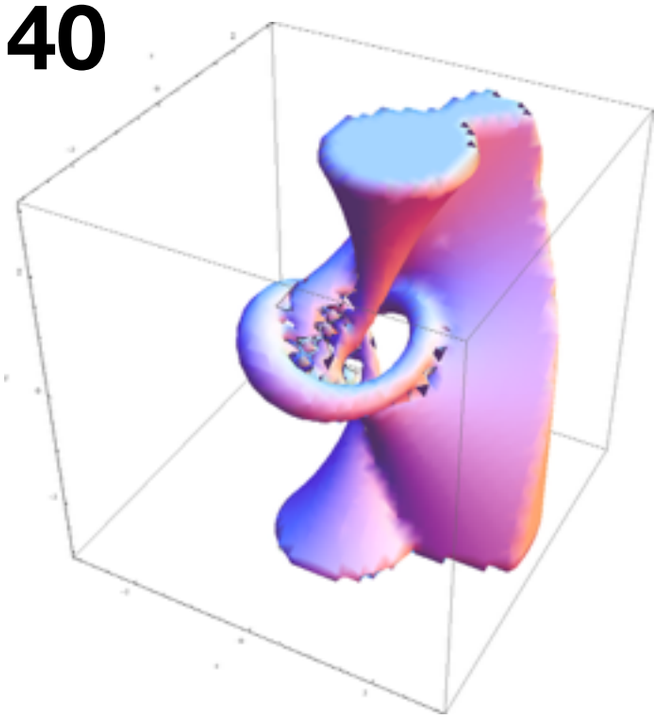
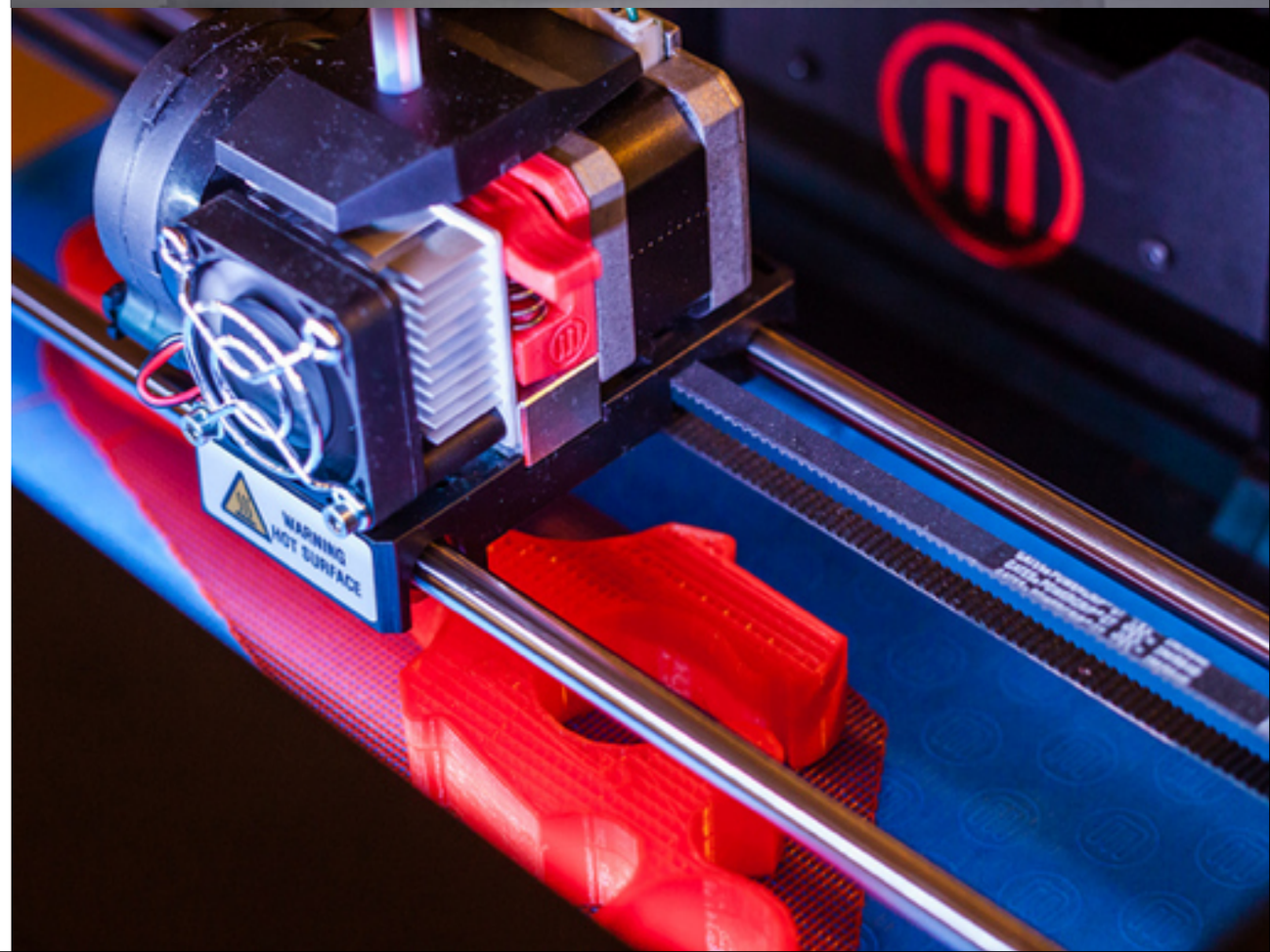


Generating 3D Models—Trefoil Knot + Page.
Created in Mathematica.

3D Printing.

- “Process of making a three-dimensional solid object of virtually any shape from a digital model.” -Wikipedia
- **Makerbot** Replicator 2
- **Additive Layering**: printer extrudes melted plastic in thin (100 micron, 0.1 millimeter) layers and builds object from

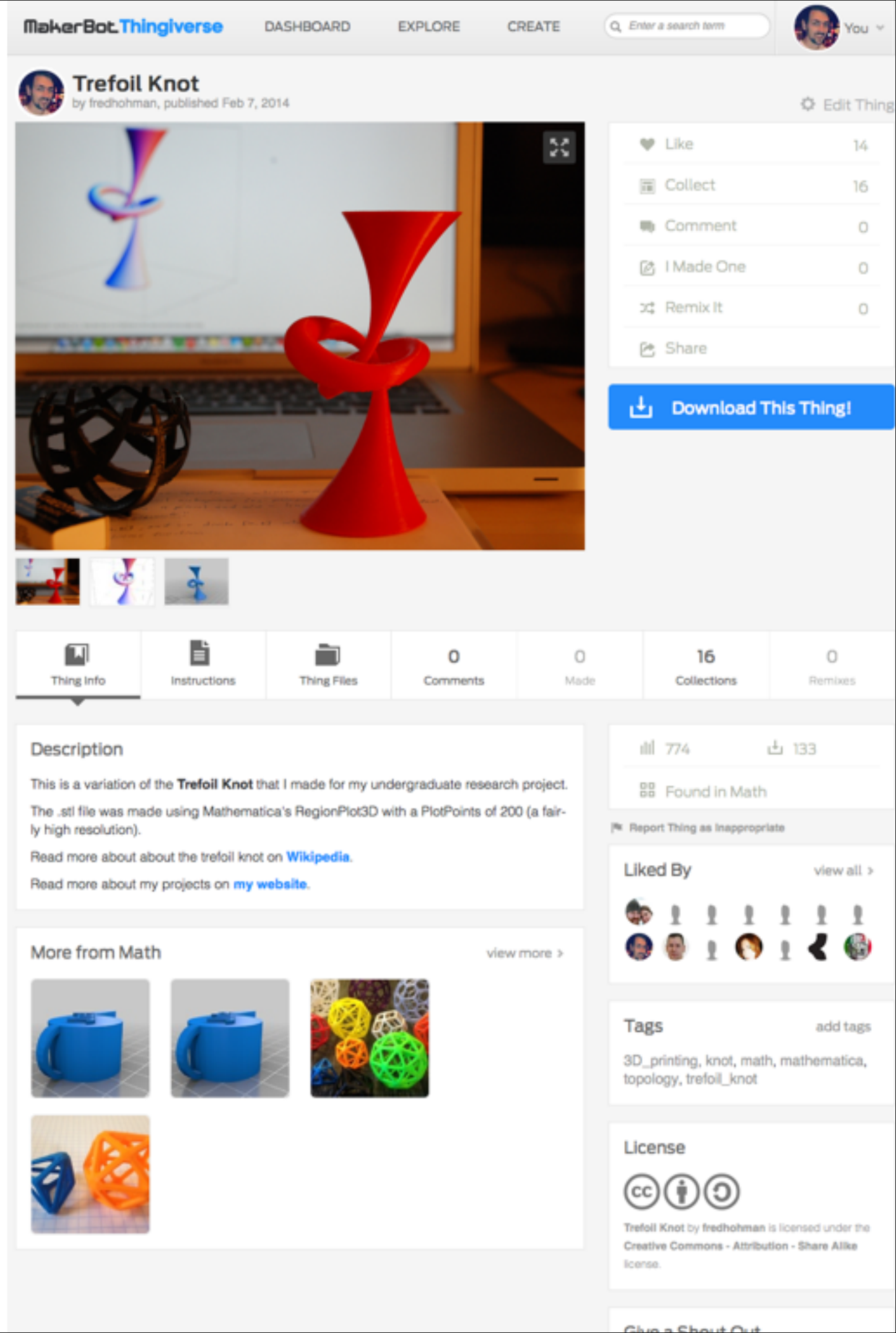


3D Printing.

- Controls:
  - **Rafts** and **Supports**
  - Infill percentage (how solid/hollow the object is)
  - Layer Height (100 microns, 0.1 millimeters)
  - Extruder temperature (~230° C, ~446° F)
  - Extruder speed when printing/traveling (90 mm/s, 150 mm/s)

Results and Future Research.

- Posted on **Thingiverse**
  - Makerbot's website for digital design file sharing
  - Currently has ___ views and ___ downloads
- Determining best way to cut pages to build knots
- Print more complicated knots with more pages



Final Remarks and Questions.

- **Tinkercad**: free 3D modeling program that runs in the browser (Chrome)—no downloading required!
    - Short tutorial (~10 minutes)
    - Easiest way to begin to model
- **Thingiverse**
- 3D printing technology is rapidly growing

Final Remarks and Questions.

Questions?

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References. (online)

- Mandalland Blogspot—Triangles
- Wikipedia—3D Printing
- Thingiverse—Stereographic Projection
- Makerbot
- Atomic Spin—Replicator 2 Issues